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A PRELIMINARY REPORT ON NUCLEAR ROCKET DYNAMICS AND CONTROL

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OF THE UNIVERSITY OF CALIFORNIA LOS ALAMOS NEW MEXICO

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A PRELIMINARY REPORT ON NUCLEAR ROCKET DYNAMICS AND CONTROL

by

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ABSTRACT

Collection of Memoranda in this report

(1) N-4-676U Preliminary Report on Nuclear Rocket Engine Control Systems

(2) N-4-703U A Mathematical Model to Represent the Basic Dynamics of a Nuclear Rocket System

The reader should also be referred to "Nuclear Rocket Engine Control" by R. R. Mohler & J. E. Perry, Jr., Nucleonics (McGraw-Hill publication), Vol. 19, No. 4 (April, 1961)

I. PRELIMINARY REPORT ON NUCLEAR ROCKET ENGINE CONTROL SYSTEMS

Date written: June 8, 1960

INTRODUCTION

The basic thrust equations are presented here to show the functional dependence of pressure, temperature, and flow rate at the core exit on the engine thrust. Possibleloop configurations of control systems are presented. The transducers involved and need for further study and possible development should be emphasized.

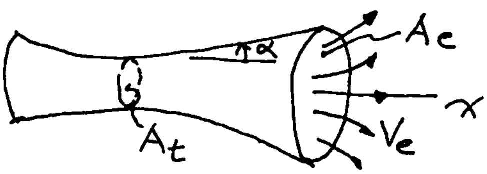
DISCUSSION

Most of the equations and transfer functions have been treated before in the analysis of Kiwi-A and Kiwi-B[1]. In addition the following equations are of interest[2].

Thrust can be considered as a summation of momentum thrust and pressure thrust

$$F = \rho_e V_{ex}^2 A_e + (p_e - p_a) A_e$$

but $\dot{W} = \rho_e g V_{ex} A_e \quad \therefore \quad F = \frac{\dot{W}}{g} V_{ex} + (p_e - p_a) A_e$



$$V_{ex} \approx \frac{1}{2}(1+\cos\alpha) V_e$$

The ideal exit velocity

$$V_e' = \left\{ 2(g)\left(\frac{\gamma}{\gamma-1}\right) R T_c \left[1 - \left(\frac{p_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\frac{1}{2}}$$

and the actual exit velocity

$$V_e = \phi V_e' \qquad \phi \approx 0.9 \text{ to } 1.0$$

Therefore the thrust can be defined

$$F \approx \psi \sqrt{T_c}\,\dot{W} \approx I_{sp}\,\dot{W} \qquad (1)$$

where,

$$p_e \approx p_a$$

$$\psi = \Phi\lambda\left\{g^{-1}\left(\frac{\gamma}{\gamma-1}\right)R\left[1-\left(\frac{p_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} \approx \text{constant}$$

$$\lambda = \tfrac{1}{2}(1+\cos\alpha)$$

Perhaps a simpler and more useful relation for our purposes is

$$F = C_f\,P_c\,A_t \qquad (2)$$

where

$$C_f = \lambda C_d \Phi \Omega \left(\frac{2\gamma}{\gamma-1}\right)^{1/2}\sqrt{Z_t} + \left(\frac{p_e - p_a}{P_c}\right)\frac{A_e}{A_t}$$

$$\Omega = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$Z_t = 1 - \left(\frac{p_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}$$

$$C_f \approx \text{constant}$$

Thus it is seen that the stagnation pressure at the core exit is the one variable that most strongly effects engine thrust. The temperature effect is seen as a variation in the ratio of specific heats as well as its effect on pressure. Thrust coefficients for particular nozzle designs (C_f vs. P_c/p_e and Ae/A_t) are available[2].

Since durable, fast, and accurate pressure transducers are available, one could expect to control engine thrust by controlling this pressure. This method has been used on rockets that are operational with good resolution and reliability.

It is of interest to note from equation (1) that specific impulse increases as the square root of temperature, thus showing the desirability of a high core outlet temperature; also that specific impulse is high for a low molecular weight ($R = \frac{1545}{\overline{m}}$), which shows the desirability of hydrogen propellant and nuclear rocket engines.

By considering the evolution of the Kiwi-A reactors and the above equations, a logical choice of a flyable control system is shown in figure 1[3,4]. The programmer on the left puts out a thrust demand (or core outlet pressure demand). This demand is compared with the measured thrust (or pressure). The thrust error signal is used to properly adjust the thrust in a desired manner. The flow loop could be closed to speed the loop response and help smooth out the unknowns and changes in the flow characteristics with operating conditions. But if the flow is not measured or calculated accurately it may be advisable not to close this loop. In this case the power demand is scheduled from the flow demand according to the temperature desired for start up. At rated conditions this scheduler appears as a constant. The power loop is closed. It is advisable to close either the power loop or temperature loop for safety reasons as well as unknowns and changes in the open loop dynamics.

If a good temperature measurement or calculation can be made it is desirable to feed back temperature. This could be fed back to vary the scheduler gain. However if a good temperature measurement is available, the temperature loop could replace the power loop. This seems especially feasible if the power measurements cannot be improved and the flyable systems have as fast a core thermal time constant as is anticipated.

A system which is conceivable in the distant future is shown in figure 2[3]. This system assumes that a poison rod is adjusted previous to the operation. This rod (or rods) is used to take care of the unknowns in reactivity contributions. The loop could be closed through this rod. to control slow transients. The major loop and fast transients are controlled by the propellant reactivity. A simulation of a simplified model of this type has been made on the analog computer. The system was stable and was made to respond favorably with a simple compensation.

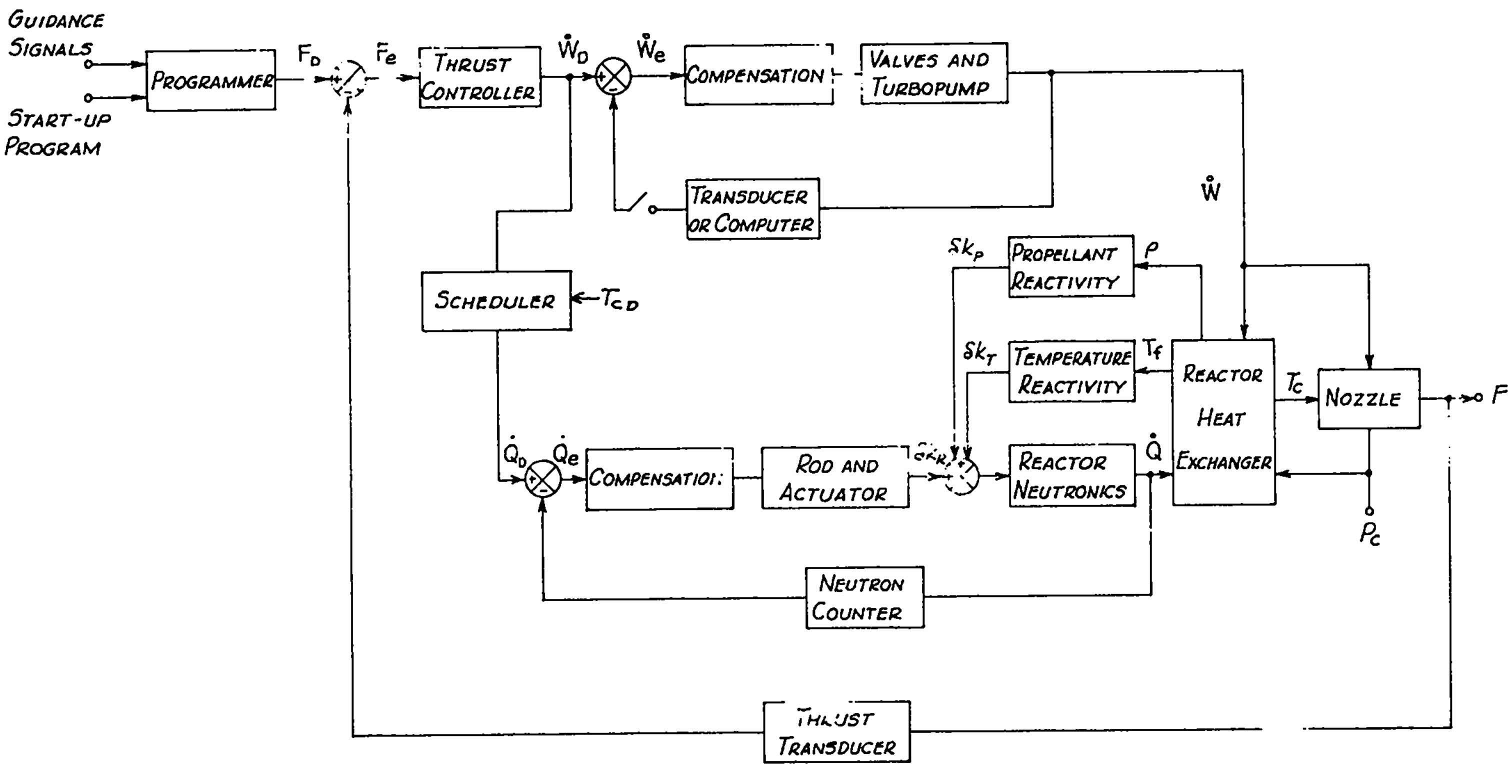


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Fig 1 Functional Block Diagram of a Possible Conventional Control System for Nuclear Rocket Propulsion.

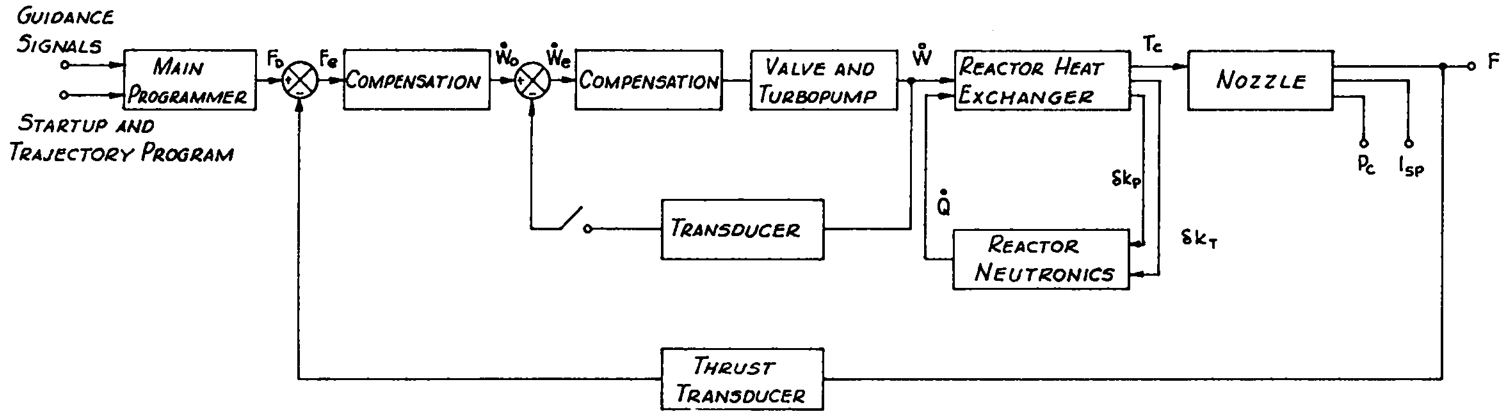


Fig. 2 Functional Block Diagram of a Simple Nuclear Rocket Propulsion Control System with no Moving Rods.

CONCLUSIONS

In a flyable nuclear rocket engine control system the thrust loop is most conveniently closed by closing the loop from pressure measured at the nozzle input. Either the power loop or the temperature loop should be closed. The selection would be strongly dependent on the transducers available as well as accident considerations. It may be desirable but not necessary to close the flow loop. This decision would also depend on the transducers available. Further investigations, studies and possible development of tranducers should certainly be encouraged. The main loop which involves a pressure transducer seems to have the fewest problems. But an investigation as to the state of the art should be made to bring us up to date. Temperature, power and flow transducers need a great deal of work. Even though only one of the first two may be used for automatic control purposes, the others are desirable for making decisions and manual operations during a test.

Notation added 10/7/60:

It is doubtful that pressure transducers will be accurate enough to control rocket velocity to the precision desired. Therefore it may be necessary to use accelerometers to control velocity. The pressure or thrust could still be controlled as an inner loop of the main velocity loop: A more detailed study of a flyable system is presently being made.

RRM

REFERENCES

(1) LAMS-2539, Vol. I, R. Mohler, to be issued
(2) Zucrow - Aircraft and Missile Propulsion
(3) LAMD 1636, R. Mohler
(4) N-4-596U, H. Demuth and R. Mohler (Not available)

GLOSSARY

SYMBOL	DESCRIPTION	UNITS
A_e	Nozzle Exhaust Area	ft^2
A_t	Nozzle Throat Area	ft^2
A_v	Value Flow Area	ft^2
C_d	Nozzle Coefficient of Discharge	
C_f	Thrust Coefficient	
F	Thrust	#
g	Acceleration from Gravity	ft/sec^2
I_{sp}	Specific Impulse	sec
Sk	Total Reactivity	
Sk_p	Propellant Reactivity	
Sk_T	Temperature Reactivity	
$\bar{m}$	Molecular Weight	# mass/# mole
P_c	Stagnation Pressure at Core Exit	$\#/ft^2$
p_c	Pressure at Core Exit	$\#/in^2$
P_e	Pressure at Nozzle Exit	$\#/in^2$
P_a	Atmospheric Pressure	$\#/in^2$
$\dot{Q}$	Reactor Power	BTU/sec
R	Gas Constant	$ft/^{o}R$
T_c	Temperature at Core Exit	^{o}R
T_f	Average Temperature of Fuel & Moderator	^{o}R
t	Time	sec
V_e	Ideal Exhaust Velocity	ft/sec
V_e'	Actual Exhaust Velocity	ft/sec

SYMBOL	DESCRIPTION	UNITS
V_{ex}	Axial Component of Exhaust Velocity	ft/sec
$\dot{W}$	Propellant Weight Flow Rate	#/sec
Z_t	A pressure constant	

GREEK SYMBOLS

α	Nozzle Divergence Angle	
γ	Ratio of Specific Heats	
λ	Nozzle Divergence Coefficient	
ρ	Mass Density	$slug/ft^3$
ϕ	Velocity Coefficient	
ψ	Flow Factor	
Ω	Constant of Specific Heat Ratios	$sec/{}^{\circ}R^{\frac{1}{2}}$

II. A MATHEMATICAL MODEL TO REPRESENT THE BASIC DYNAMICS OF A NUCLEAR ROCKET SYSTEM*

Date written: August 12, 1960

The rocket reactor considered is a direct heat exchange cycle, two pass system. Here the hydrogen propellant enters and cools the nozzle and reactor reflector, then passes on through the reactor core where it is heated to exhaust temperature. The propellant exhausts from the reactor through a nozzle which is choked under normal operating conditions.

A turbo pump supplies liquid hydrogen to the heat exchanger from a constant pressure tank. A pair of valves supply gas from a constant pressure source to the turbine of the turbo-pump. Thus variation of the valve stems produces a variation in liquid hydrogen flow to the heat exchanger. The dynamic characteristics of the turbo pump and associated feed lines are considered in this model.[1]

The dynamic characteristics of the reactor core are treated by non linear space independent differential equations. The dynamic heat transfer characteristics of the core are treated by means of a lumped parameter model such that the resulting differential equations are space independent. The accuracy under dynamic conditions of the lumped parameter model is a function of the number of lumps into which the core is broken. Thus a very large number of lumps would yield very high accuracy. However the representation rapidly approaches reasonable accuracy with few lumps. The propellant pressure drop within the core is generated as a function of reactor power level and propellant flow rate. This pressure drop is lumped in the same manner as the heat exchanger such that the space variable is eliminated.

The six delayed neutron group, space independent, differential equations are used to represent the neutron kinetics. The total reactivity variation is represented as the sum of the following items: (1) The negative reactivity contribution of the control rods. (2) Variation in reactivity due to the thermal expansion of the reactor which varies the neutron non-leakage probability. Also included in this category is the change in thermal energy of the neutrons which through the associated fission cross-section affects the power level, (3) The reactivity contribution of the hydrogen propellant due to density (which changes at various positions in the core). This reactivity contribution is of course due to the moderating properties of the propellant.

The propellant exhaust nozzle is assumed choked in this model. Isentropic pressure relations, modified by an efficiency factor, are used in the nozzle representation.

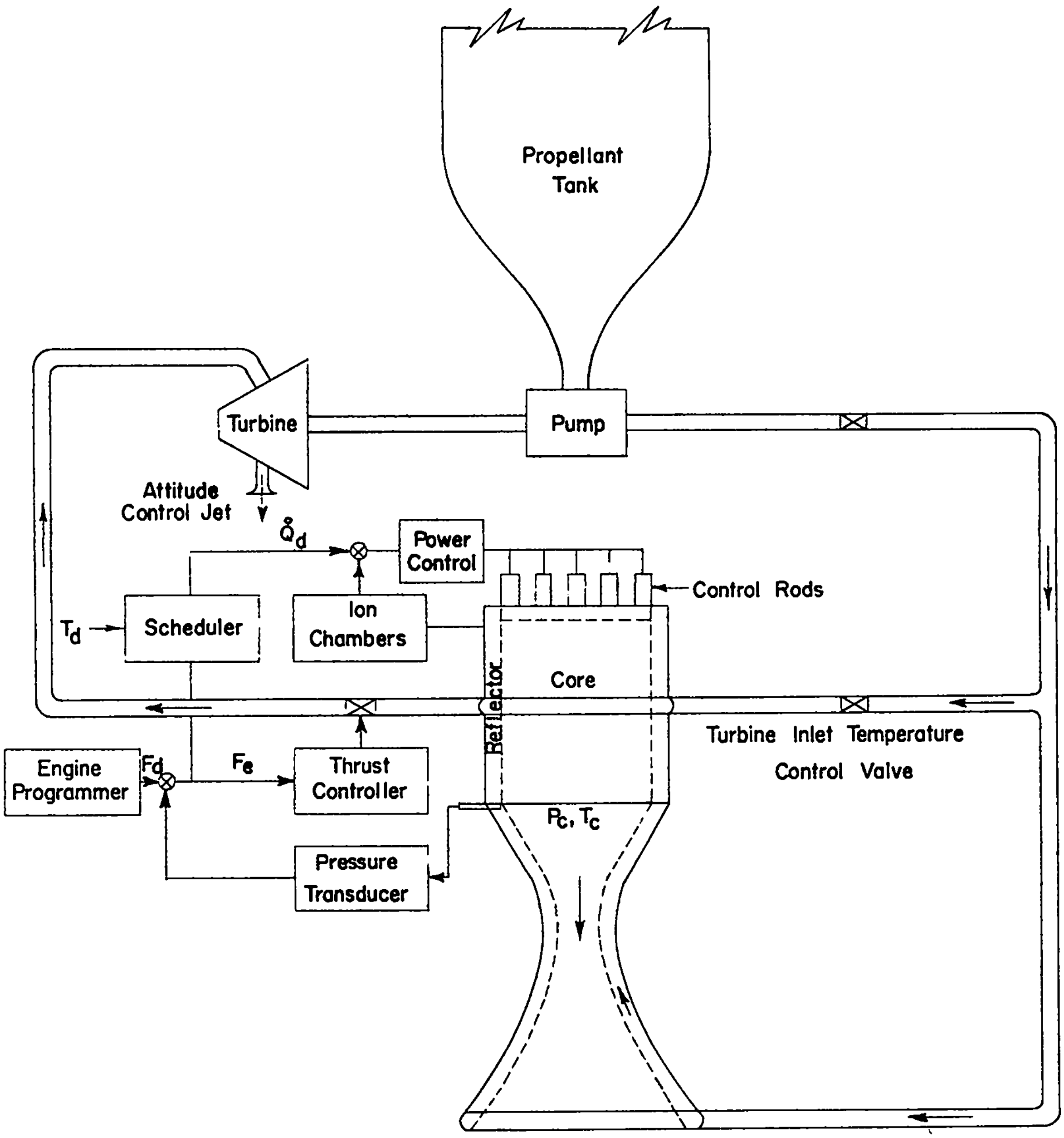
The basic application of this mathematical model is to the development of nuclear rocket engine controls. The previous equations are linearized about steady state conditions to obtain transfer functions. The dynamic stability of the system is determined from these transfer functions. It is seen that the stability of the power loop is actually improved by the addition of the propellant reactivity and is dynamically stable even with a zero temperature coefficient of reactivity. However in this latter case the control rod poison initially present must be at least equal to the amount of poison required at some operating condition. If enough poison is not initially present the temperature will approach a steady state value which may be above design value.

The thermal time constant is seen to be dependent on the core mass heat capacity and the flow rate. The thermal time constant of a typical second stage nuclear rocket could be about one second or less. For very fast cores this time constant could be several milliseconds. The neutronics band width is β/ℓ^*, where β = .0065 for U^{235} and ℓ^* could be several microseconds. The turbopumps inertial time constant is in the order of a second.

Several control loop schemes are shown. The thrust and temperatue (or specific impulse) are of prime interest. But due to present limitations on transducers it may be necessary to control temperature by scheduling power from flow rate or another variable in the main thrust loop. The temperature transient during startup must be such as to consider limitations due to thermal stresses and yet meet the need for maximum specific impulse.

(1) Footnote - The turbopump characteristics are based on information supplied by the Rocketdyne Division of North American Aviation.

* Presented at the Winter Meeting of the American Nuclear Society, San Francisco, California, December, 1960.



Schematic Diagram of a Nuclear Rocket Engine System

DEFINITION OF TERMS
USED IN MODEL OF REACTOR
AND NOZZLE

Symbol	Description	Units
A	Heat transfer area	in^2
A_t	Nozzle throat area	in^2
A_p	Effective core exit area (Fluid flow)	in^2
C	Specific heat of fuel-moderator	$Btu/lb_m\,°R$
C_d	Nozzle coefficient of discharge	dimensionless
C_f	Thrust coefficient	dimensionless
C_i	Heat due to the density of ith delayed-neutron-group precursor	Btu
C_p	Specific heat at constant pressure of propellant	$Btu/lb_m\,°R$
C_r	Propellant density coefficient or reactivity	lb_m^{-1}
C_t	Temperature coefficient of reactivity	$°R^{-1}$
d_1	Diameter of flow passage	in
d_2	Effective outer diameter fuel-moderator element	in
F	Thrust	lb_f
f	Friction factor	dimensionless
g	Factor for conversion from English Absolute to English Gravitational System of units	$lb_m in/lb_f\ sec^2$
h	Total heat transfer coefficient	$Btu/in^2\ sec\ °R$
h_c	Convective heat transfer coef--ficient	$Btu/in^2\ sec\ °R$
h_m	Effective conduction heat transfer coefficient	$Btu/in^2\ sec\ °R$

K	Propellant thermal conductivity	Btu/in sec °R
K_m	Conductivity of fuel moderator	Btu/in sec°R
K_t	Core weighting factor	dimensionless
δK	Total reactivity	dimensionless
δK_p	Propellant reactivity	dimensionless
δK_r	Control rod reactivity	dimensionless
δK_t	Temperature reactivity	dimensionless
ℓ	Length of core	in
ℓ^*	Mean effective neutron lifetime	sec
M	Mach number of propellant	dimensionless
M_c	Mach number of propellant at core exit	dimensionless
P	Propellant pressure	lb_f/in^2
P_c	Propellant stagnation pressure at core exit	lb_f/in^2
P_{cs}	Propellant static pressure at core exit	lb_f/in^2
Pr	Prandtl number	dimensionless
$\dot{Q}$	Reactor power	Btu/sec
R	Specific gas constant	lb_f in/lb_m °R
Re	Reynolds number	dimensionless
S	Laplace transform variable	sec^{-1}
T_{amb}	Ambient temperature	°R
T_c	Propellant temperature at core exit	°R
T_{ca}	Average core temperature	°R
T_f	Propellant film temperature	°R
T_g	Average propellant temperature	°R

T_o	Outlet propellant temperature	°R
T_w	Average core wall temperature	°R
t	Time	sec
V	Velocity	in/sec
V_p	Volume of propellant	in^3
W	Mass of fuel-moderator	lb_m
$\dot{W}$	Propellant mass flow rate	lb_m/sec
$\dot{W}_c$	Propellant mass flow rate at core exit	lb_m/sec
Y	Fuel-moderator effective thickness	in
Z	Compressibility	dimensionless
β	Total delayed neutron fraction	dimensionless
β_i	ith group fraction of total neutrons from fission	dimensionless
Δ	Refers to a perturbation about a steady state condition	dimensionless
λ_i	Decay constant of ith delayed-neutron-group precursor	sec^{-1}
η	Fraction of power generated	dimensionless
μ	Viscosity of propellant	lb_f sec/in^2
ν	Number of flow passages	dimensionless
τ	Time constant	sec
π	3.141593	- -
ρ	Propellant Density	lb_m/in^3
Subscript n	Stage Number	- -

SYSTEM DYNAMICS EQUATIONS

Heat Exchanger:

$$\eta_n \dot{Q} = W_n C_n \frac{dT_{wn}}{d\tau} + A_n h_n (T_{wn} - T_{gn}) \qquad (1)$$

$$\dot{W}(H_{on} - H_{on-1}) = A_n h_n (T_{wn} - T_{gn}) = \dot{W} C_{pn} (T_{on} - T_{on-1}) \qquad (2)$$

$$T_{gn} = \frac{T_{on-1} + T_{on}}{2} \qquad (3)$$

$$h_{cn} = \frac{.036 K_n Re_n^{0.8} \; Pr_n^{0.4}}{d_1 (\frac{x}{d_1} + 1)^{0.1}} \left(\frac{T_{gn}}{T_{fn}} \right)^{0.8} \qquad (4)$$

$$R_{en} = \frac{\rho_n V_{nd}}{\mu_n} = \frac{4\dot{W}}{\mu_n \pi d_1}$$

$$T_{fn} \approx \frac{T_{gn} + T_{wn}}{2}$$

All quantities evaluated at film conditions except $\dot{W}$.

$$h_{mn} = \frac{2K_{mn}}{d_1 \log_e \frac{d_2}{d_1}} \qquad (5)$$

$$h_n = \frac{h_{cn} h_{mn}}{h_{cn} + h_{mn}} \qquad (6)$$

Neutronics :

Propellant Reactivity

$$\rho_n = \frac{P_n}{Z_n R\, T_{gn}} \tag{7}$$

$$\delta K_{pn} = C_{rn} V_{pn} \rho_n \tag{8}$$

$$\delta K_p = \sum_{n=1}^{n} \delta K_{pn} \tag{9}$$

Temperature Reactivity

$$T_{ca} = \sum_{n=1}^{n} K_{tn} T_{wn} \tag{10}$$

$$\delta K_t = C_t (T_{ca} - T_{amb}) \tag{11}$$

$$\delta K = \delta K_r + \delta K_p + \delta K_t \tag{12}$$

$$\frac{d\dot{Q}}{dt} = \frac{\delta K}{l^*}\dot{Q} - \frac{\beta \dot{Q}}{l^*} + \sum_{i=1}^{6} \lambda_i C_i' \tag{13}$$

$$\frac{dC_i'}{dt} = \frac{\beta_i}{l^*}\dot{Q} - \lambda_i C_i' \tag{14}$$

Nozzle:

$$F = C_f A_t P_c \tag{15}$$

$$P_c = \frac{\dot{W}_c \sqrt{R T_c}}{C_d \psi A_t \sqrt{2g}} \tag{16}$$

Where

$$\psi = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma - 1}} \left(\frac{\gamma}{\gamma + 1}\right)$$

$$P_c = \left[1 + \frac{\gamma - 1}{2} M_c^{2}\right]^{\frac{+\gamma}{\gamma - 1}} P_{cs}$$

$$M_c = \frac{A_t}{A_p} \left[\frac{\gamma + 1}{2\left(1 + \frac{\gamma - 1}{2} M_c^2\right)}\right]^{-\frac{\gamma + 1}{2(\gamma - 1)}}$$

Reactor Pressure Drop:

$$P_{drop\ n} = \frac{\dot{W}_{co}}{\frac{d_1}{2}\left(\frac{\pi d^2}{4}\right) g \nu} (\psi_n + f_n V_n) l_n \tag{17}$$

Where

$$M_n = \frac{V_n}{(\gamma_n g Z_n R T_{gn})^{\frac{1}{2}}}$$

$$V_n = \frac{\dot{W}_c}{\rho_n \nu \left(\frac{\pi d_1^2}{4}\right)}$$

$$f_n = \frac{1.6\, h_{cn}}{C_{pn} V_n \rho_n}$$

$$\rho_n = \frac{p_n}{Z_n R T_{gn}}$$

$$\psi_n = \frac{V_n}{(1 - M_n^2)} \left[\frac{\eta_n \dot{Q} \frac{d_1}{2}}{W_c l_n C_{pn} T_{gn}} + f_n \gamma_n M_n^2\right]$$

Transfer Functions:

$$\frac{\Delta \delta K_p}{\Delta P_n} = \frac{C_{rn} V_{pn}}{Z_n R T_{gno}} \qquad (18)$$

$$\frac{\Delta \delta K_p}{\Delta T_{gn}} = \frac{-P_{no} C_{rn} V_{pn}}{Z_n R T_{gno}^2} \qquad (19)$$

$$\frac{\Delta \dot{Q}}{\delta K} = \frac{\dot{Q}_o}{l^* S \left[1 + \sum_{i=1}^{6} \frac{\beta_i}{l^* (S + \lambda_i)} \right]} \qquad (20)$$

One lump heat exchanger

$$\frac{\Delta T_o}{\Delta \dot{Q}} = \frac{K_1}{(T_h S + 1)} \qquad (21)$$

$$\frac{\Delta T_o}{\Delta \dot{W}} = \frac{K_2 (T_2 S + 1)}{T_h S + 1} \qquad (22)$$

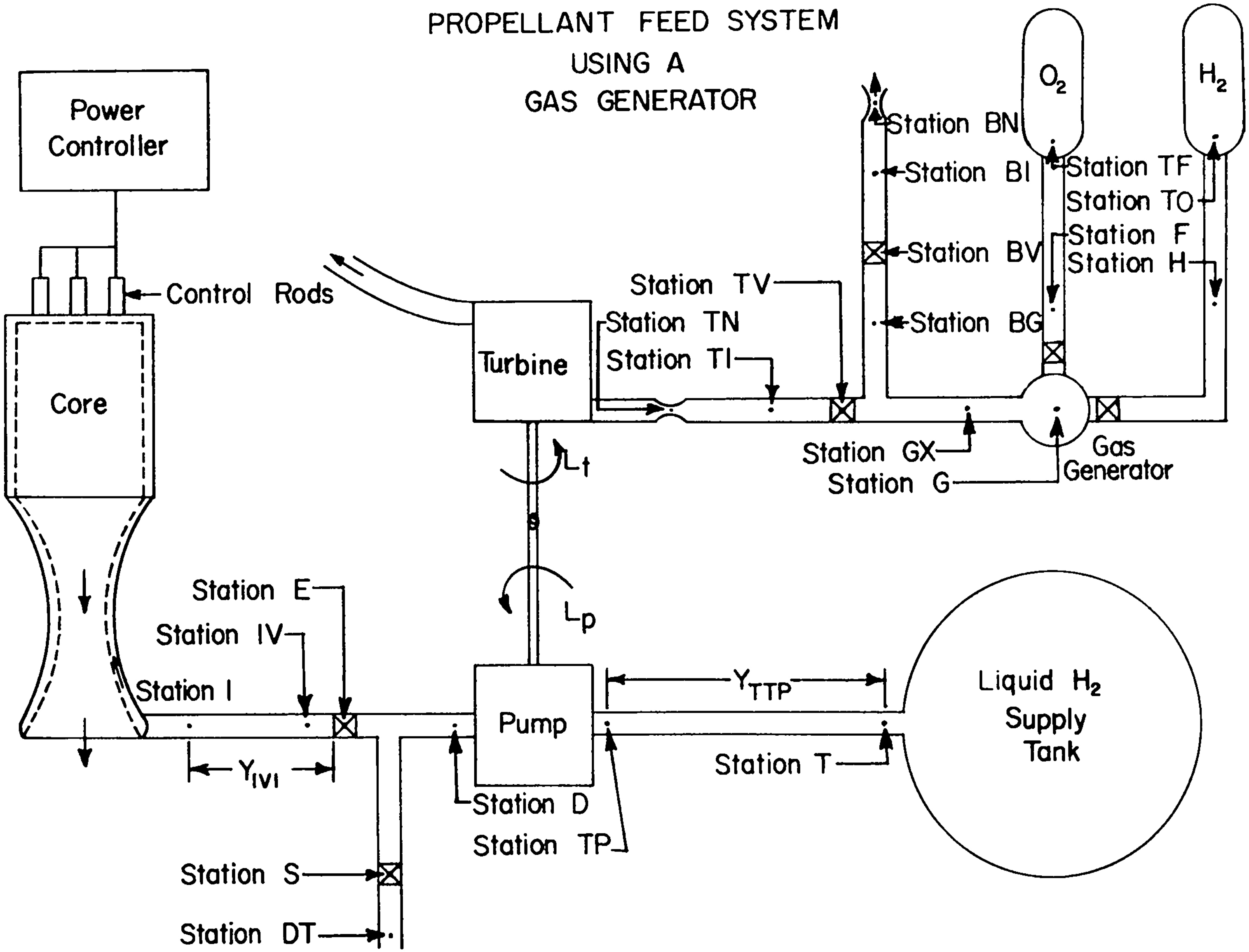
$$T_h = \frac{WC}{2 C_p \dot{W}_o} + \frac{WC}{h_o A} + \left(\frac{WCY}{KA} \right) \qquad (23)$$

$$T_2 = \frac{0.2\, WC}{h_o A}$$

$$K_1 = \frac{1}{\dot{W}_o C_p}$$

$$K_2 = \frac{-\dot{Q}_o}{\dot{W}_o^2 C_p}$$

PROPELLANT FEED SYSTEM
USING A
GAS GENERATOR



DEFINITION OF TERMS

USED IN MODEL OF HOT GAS GENERATOR

PROPELLANT FEEDSYSTEM TO REACTOR

Symbol	Description	Units
A_{BN}	Throat area of nozzle in turbine bypass line	in^2
A_{BV}	Valve throat area in turbine bypass line	in^2
$A_{BV\ MAX}$	Maximum valve throat area in turbine bypass line	in^2
A_e	Valve throat area in pump discharge line	in^2
A_F	Valve throat area in liquid oxygen supply line to gas generator	in^2
A_H	Valve throat area in liquid hydrogen supply line to gas generator	in^2
A_S	Valve throat area in reactor bypass line	in^2
A_{TN}	Turbine nozzle area	in^2
A_{TV}	Valve throat area in turbine supply line	in^2
$A_{TV\ MAX}$	Maximum valve throat area in turbine supply line	in^2
d_{IVI}	Diameter of liquid propellant reactor supply line (between stations IV and I)	in
d_{TTP}	Diameter of liquid propellant pump supply line (between stations T and TP)	in
f_{IVI}	Friction coefficient in liquid propellant reactor supply line	dimensionless
f_{TTP}	Friction coefficient in liquid propellant pump supply line	dimensionless

Symbol	Description	Units
g	Factor for conversion from English Absolute to English Gravitational System of Units	$\frac{lb_m in}{lb_f sec^2}$
I	Turbopump polar moment of inertia	$in\ lb_f sec^2$
K	An empirical constant	
L_P	Pump torque	$in\ lb_f$
L_T	Turbine torque	$in\ lb_f$
N	Rotational speed of turbo-pump	$\frac{rev}{min}$
P_1	Pressure at inlet to cooling passage in nozzle	$\frac{lb_f}{in^2}$
P_{IV}	Pressure on reactor side of the valve in the pump discharge line	$\frac{lb_f}{in^2}$
P_{BI}	Pressure upstream of nozzle in turbine bypass line	$\frac{lb_f}{in^2}$
P_D	Pressure at pump discharge	$\frac{lb_f}{in^2}$
P_{DT}	Exhaust pressure of liquid propellant in reactor bypass line	$\frac{lb_f}{in^2}$
P_G	Pressure in gas generator	$\frac{lb_f}{in^2}$
P_{rio}	Pressure of propellant at inlet to core reflector	$\frac{lb_f}{in^2}$
P_T	Propellant dewar pressure	$\frac{lb_f}{in^2}$
P_{TO}	Pressure in liquid hydrogen supply tank for gas generator	$\frac{lb_f}{in^2}$
P_{TI}	Pressure upstream of nozzle in turbine supply line	$\frac{lb_f}{in^2}$

Symbol	Description	Units
P_{TF}	Pressure in liquid oxygen supply tank for gas generator	$\frac{lb_f}{in^2}$
P_{TP}	Pressure of propellant at inlet to pump	$\frac{lb_f}{in^2}$
R_G	Gas constant for gases in gas generator	$\frac{in\ lb_f}{lb_m\ {}^oR}$
T_{OG}	Total temperature in the gas generator	${}^oR$
V_G	Volume of the gas generator and pipe line from the gas generator to turbine and turbine bypass valves	in^3
W_I	Propellant flow at inlet to cooling passage in nozzle	$\frac{lb_m}{sec}$
W_{IV}	Propellant flow on the reactor side of the valve in the pump discharge line	$\frac{lb_m}{sec}$
$\dot{W}_{BN}$	Hot gas flow at nozzle throat in turbine bypass line	$\frac{lb_m}{sec}$
$\dot{W}_{CO}$	Propellant flow at inlet to core reflector	$\frac{lb_m}{sec}$
$\dot{W}_D$	Propellant flow at pump discharge	$\frac{lb_m}{sec}$
$\dot{W}_F$	Flow of liquid oxygen at station F in supply line to gas generator	$\frac{lb_m}{sec}$
$\dot{W}_{GX}$	Flow of hot gas at exit of gas generator	$\frac{lb_m}{sec}$
$\dot{W}_H$	Flow of liquid hydrogen at station H in supply line to gas generator	$\frac{lb_m}{sec}$
$\dot{W}_S$	Propellant flow bypassing the reactor	$\frac{lb_m}{sec}$
$\dot{W}_{TN}$	Hot gas flow through throat of the turbine nozzle	$\frac{lb_m}{sec}$

Symbol	Description	Units
X	Position of stem of turbine bypass control valve	in
X_{MAX}	Maximum travel position of stem of turbine bypass control valve	in
Y_{IVI}	Equivalent length of liquid propellant supply line to reactor	in
Y_{TTP}	Equivalent length of liquid propellant supply line to pump	in
γ	Ratio of specific heats of hot gas from gas generator	dimensionless
ρ_{IVI}	Density of liquid propellant in supply line to reactor	$\frac{lb_m}{in^3}$
ρ_D	Density of liquid propellant at pump discharge	$\frac{lb_m}{in^3}$
ρ_F	Density of liquid oxygen at station F in supply line to gas generator	$\frac{lb_m}{in^3}$
ρ_H	Density of liquid hydrogen at station H in supply line to gas generator	$\frac{lb_m}{in^3}$
ρ_{TTP}	Density of liquid propellant in supply line to pump	$\frac{lb_m}{in^3}$

Propellant Feed System Equations

1) $P_{rio} = P_I$

2) $\dot{W}_{CO} = \dot{W}_I$

3) $\dot{W}_{IV} = A_e \sqrt{2g\rho_D} \left[P_D - P_{IV} - \left(\frac{16 f_{IVI} y_{IVI}}{\pi^2 g d_{IVI}^5 \rho_{IVI}} \right) \dot{W}_{IV}^2 \right]^{\frac{1}{2}}$

4) $\dot{W}_S = A_S \sqrt{2g\rho_D} \left[P_D - P_{DT} \right]^{\frac{1}{2}}$

5) $\dot{W}_D = \dot{W}_S + \dot{W}_{IV}$

6) $P_{TP} = P_T - \left(\frac{16 f_{TTP} y_{TTP}}{\pi^2 g d_{TTP}^5 \rho_{TTP}} \right) \dot{W}_D^2$ No Waterhammer

7) $P_D = P_{TP} + 12 \rho_{TTP} N^2 \left[\psi' \left(\frac{\dot{W}_D}{N} \right) \right]$

8) $\psi' \left(\frac{\dot{W}_D}{N} \right) = K_1 + \frac{K_2}{K_C} \left(\frac{\dot{W}_D}{N} \right) + \frac{K_3}{K_C^2} \left(\frac{\dot{W}_D}{N} \right)^2$

9) $\phi' \left(\frac{\dot{W}_D}{N} \right) = K_{10} + \frac{K_{11}}{K_C} \left(\frac{\dot{W}_D}{N} \right) + \frac{K_{12}}{K_C^2} \left(\frac{\dot{W}_D}{N} \right)^2$

10) $L_T = N^2 \phi' \left(\frac{\dot{W}_D}{N} \right)$

11) $N = \frac{60}{2\pi I} \int (L_T - L_P) \, dt$

12) $L_T = K_{20} \dot{W}_{TN} - K_{21} \dot{W}_{TN} N - K_{22}$

13) $\dot{W}_{TN} = \frac{A_{TN} P_{TI}}{T_{OG}^{\frac{1}{2}}} \left[\frac{\gamma g}{R_G} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}} \right]^{\frac{1}{2}}$

14) $P_G = \frac{R_G T_{OG}}{V_G} \int (\dot{W}_H + \dot{W}_F - \dot{W}_{GX}) \, dt$

15) $\dot{W}_{GX} = \dot{W}_{TN} + \dot{W}_{BN}$

16) $$\dot{W}_H = A_H \sqrt{2g\rho_H}\left[P_{TO} - P_G\right]^{\frac{1}{2}}$$

17) $$\dot{W}_F = A_F \sqrt{2g\rho_H}\left[P_{TF} - P_G\right]^{\frac{1}{2}}$$

18) $$\dot{W}_{BN} = \frac{A_{BN} P_{BI}}{T_{OG}^{\frac{1}{2}}}\left[\frac{\gamma g}{R_G}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{\frac{1}{2}}$$

19) $$P_{BI} = \frac{P_G}{\left[\frac{1}{2} + \frac{1}{2}\sqrt{1 + 2(\gamma-1)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{A_{BN}}{A_{BV}}\right)^2}\right]^{\frac{\gamma}{\gamma-1}}}$$

20) $$K_{30} = \left(\frac{A_{TV}}{A_{TVmax}} + K_{31}\right)\left(\frac{A_{BV}}{A_{BVmax}} + K_{31}\right)$$

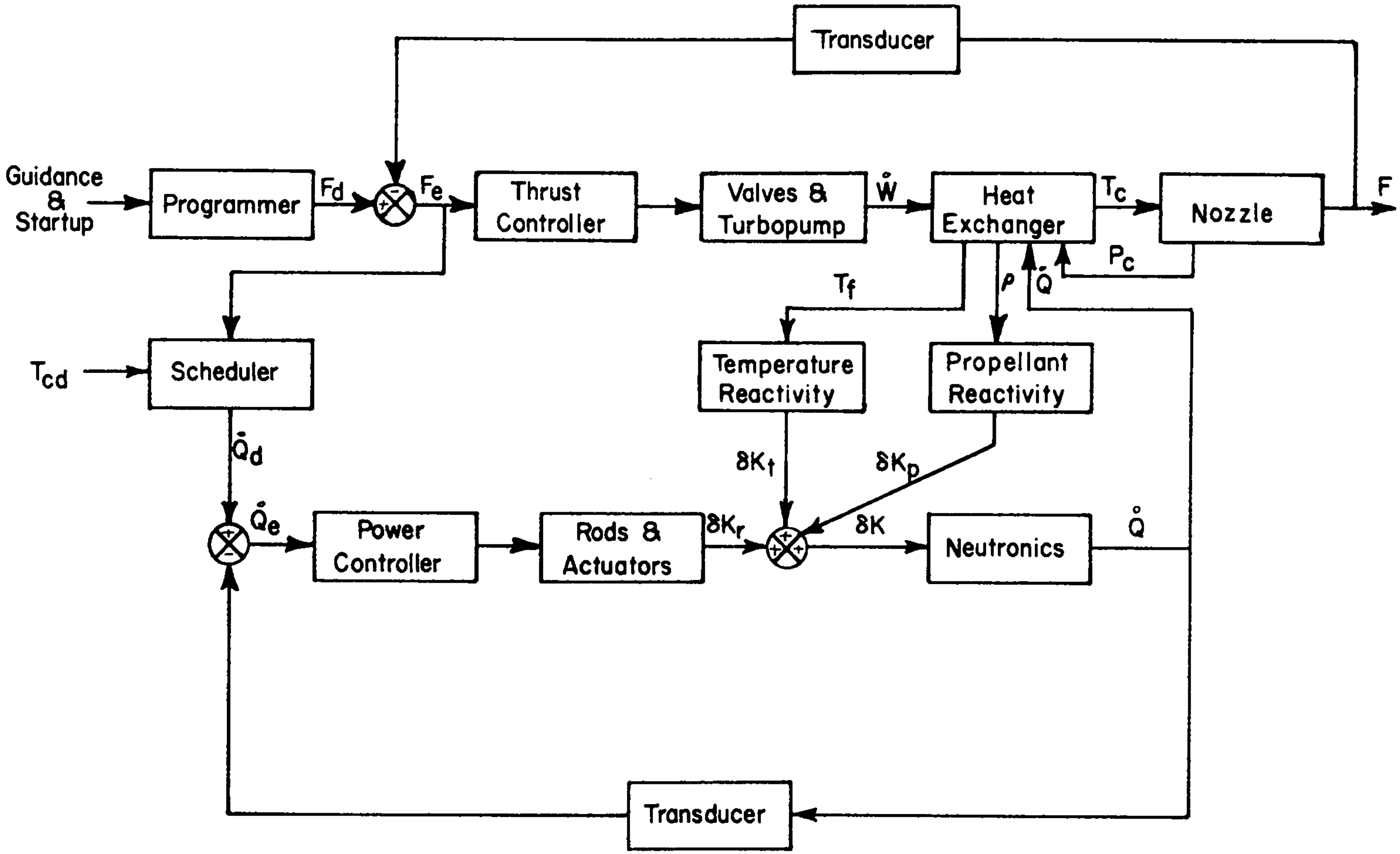
21) $$P_{TI} = \frac{P_G}{\left[\frac{1}{2} + \frac{1}{2}\sqrt{1 + 2(\gamma-1)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{A_{TN}}{A_{TN}}\right)^2}\right]^{\frac{\gamma}{\gamma-1}}}$$

22) $$A_{BV} = A_{BVmax}\left(1 - \frac{X}{X_{max}}\right)$$

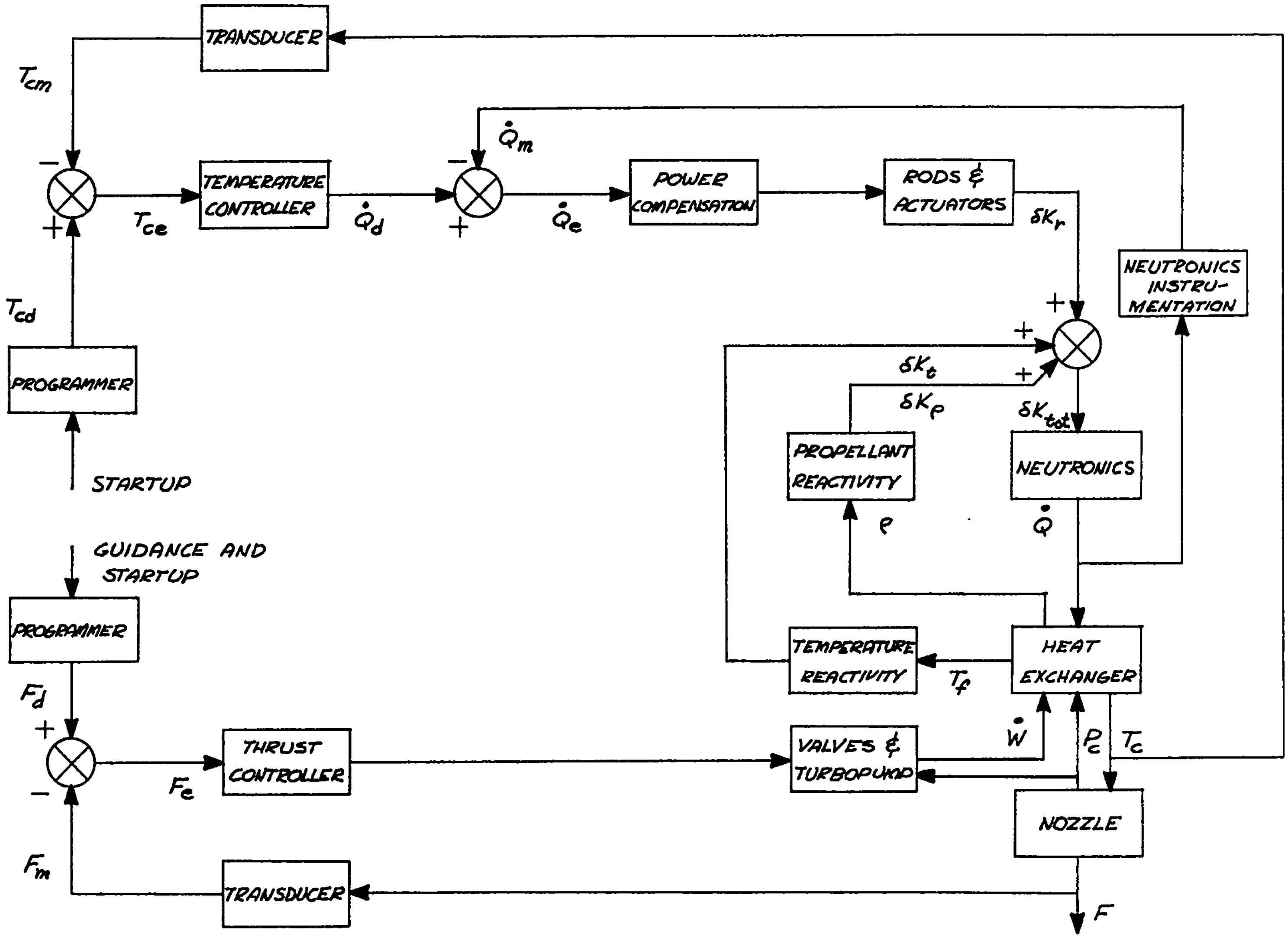
23) $P_{IV} = P_I$

24) $\dot{W}_{IV} = \dot{W}_I$

(23–24) No Waterhammer



ENGINE CONTROL SYSTEM FUNCTIONAL BLOCK DIAGRAM



FUNCTIONAL BLOCK DIAGRAM OF ENGINE CONTROL SYSTEM NO. 2